Midterm Review Problems

I. Unit Conversion and Dimensional Analysis
II. Ohm's Law and Circuit Problems
III. Error Analysis
IV. Signals and Fourier Series
V. Instrumentation, Calibration, and Resolution
VI. Probability and Statistical Analysis

I. Units and Dimensional Analysis, Beckwith Chapter 2, Section 1 of Class Notes

1) Express the universal gas constant of

$$R_u = 1545.34 \frac{ft-lbf}{^oR-(lbm-mol)}$$

In SI units

Solution:

$$R_u = \frac{1545.34 \frac{ft-lbf}{^oR-(lbm-mol)} \times 0.3048 \frac{m}{ft} \times \frac{1}{2.204 \frac{kg}{lbf}}}{\times 1.8 \frac{^oR}{^oK} \times 2.204 \frac{lbm-mol}{kg-mol} \times 9.8067 \frac{m}{\sec^2}} =$$

$$8314.47 \frac{kg-m^2}{\sec^2-{}^oK-(kg-mol)} = 8314.47 \frac{N-m}{^oK-(kg-mol)} = 8314.47 \frac{J}{^oK-(kg-mol)}$$

Units and Dimensional Analysis

2) Show in Fundamental MKS units that the power dissipated in a Resistor has units of Watts … Prove 2 Ways …

$$\rightarrow P_R = I^2 \cdot R$$

$$\rightarrow P_R = I \cdot V$$

Quantity	Name of Unit	Symbol	Expression in terms of SI base units	Expression in terms of other units
Electrical capacitance	farad	F	$m^{-2}\ kg^{-1}\ s^4\ A^2$	C/V
Electrical charge	coulomb	C	A s	
Electrical conductance	siemens	S	$m^{-2}\ kg^{-1} s^3 A^2$	A/V
Electrical inductance	Henry	H	$m^2\ kg\ s^{-2}\ A^{-2}$	
Electrical potential	volt	V	$m^2\ kg\ s^{-3}\ A^{-1}$	W/A
Electrical resistance	ohm	w	$m^2\ kg\ s^{-3}\ A^{-2}$	V/A

Units and Dimensional Analysis

2) … Solution:

$$P_R = I^2 \cdot R \rightarrow A^2 \cdot \frac{m^2 \cdot kg}{s^3 \cdot A^2} =$$

$$\frac{kg \cdot m}{s^2} \cdot \frac{m}{\sec} = \frac{N \cdot m}{\sec} = \frac{J}{s} = Watts$$

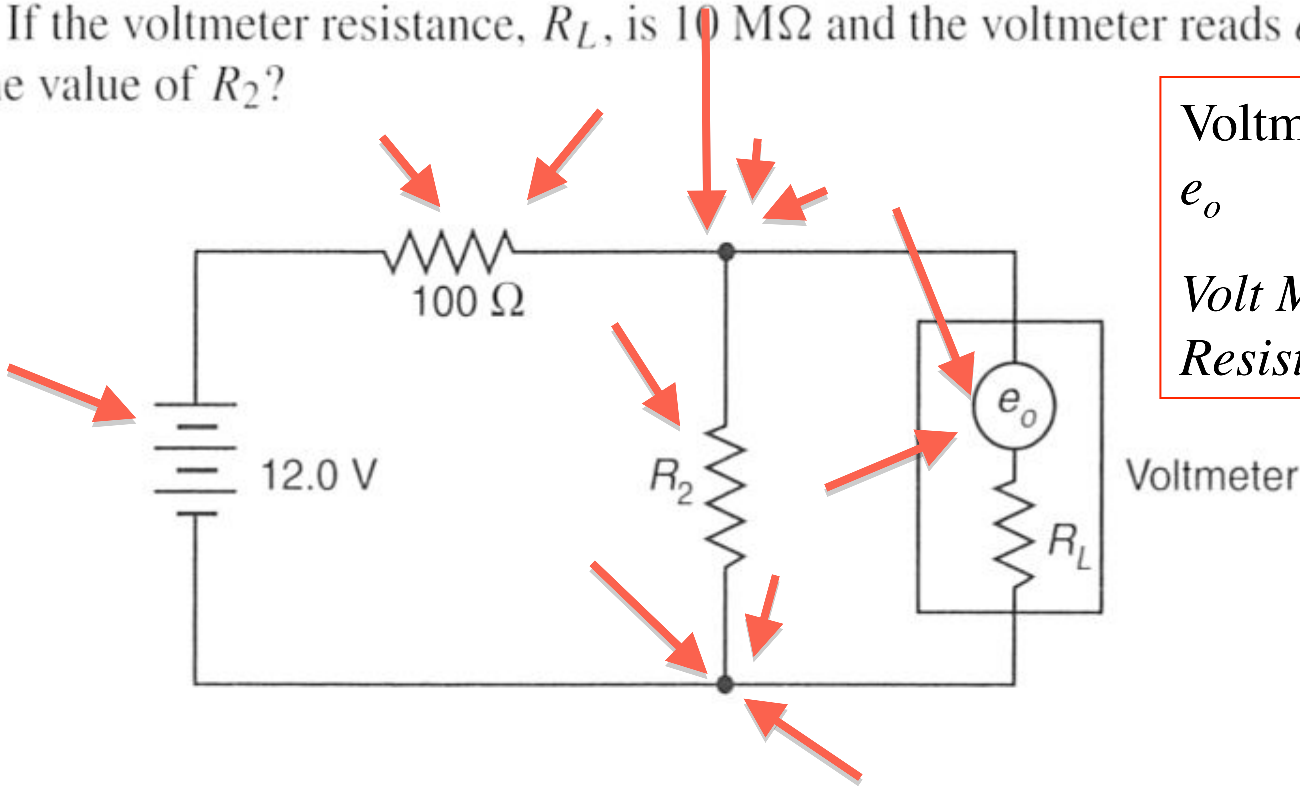
$$P_R = I \cdot V \rightarrow A \cdot \frac{m^2 \cdot kg}{s^3 \cdot A} =$$

$$\frac{kg \cdot m}{s^2} \cdot \frac{m}{\sec} = \frac{N \cdot m}{\sec} = \frac{J}{s} = Watts$$

II. Circuit Analysis, Ohms Laws, Beckwith Chapter 7, Section 2 of Class Notes

3) … Problem 7.6 in Beckwith

The circuit shown Below is used to determine the value of the unknown resistance R_2. If the voltmeter resistance, R_L, is 10 MΩ and the voltmeter reads $e_o = 4.65$ V, what is the value of R_2?



Voltmeter Output: e_o

Volt Meter Resistance: R_L

Circuit Analysis, Ohms Laws

3) … Problem 7.6 in Beckwith

Solution for Classical
Voltage Divider

$$\frac{e_o}{e_i} = \frac{R_2/100}{1 + R_2/100} = \frac{R_2}{R_2 + 100} = \frac{4.65}{12}$$

Above assumes $R_L \to \infty$ R_L, is 10 MΩ

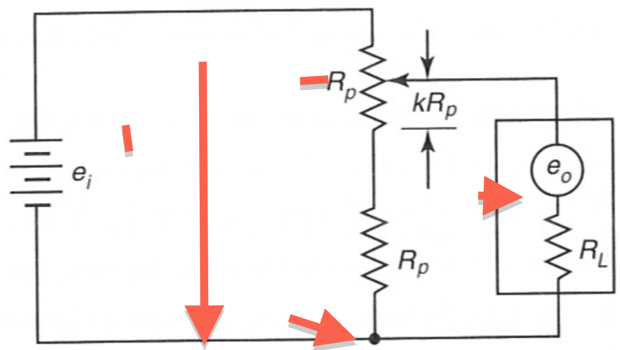
$$R_2 = 63.3\,\Omega \ll 10\,M\Omega$$

Eqn 7.5 is valid here

Circuit Analysis, Ohms Laws

Problem 7.7 in Beckwith

4) …



Voltmeter Output: e_o

Volt Meter Resistance: R_L

Determine the relationship for e_o/e_i as a function of k. Compare the results with Eq. (7.9).

What is the ratio of e_0/e_i when the meter resistance R_L becomes very large?

$$R_{eq} = (1-k)\cdot R_p + \frac{1}{\frac{1}{k\cdot R_p + R_p} + \frac{1}{R_L}} = (1-k)\cdot R_p + \frac{(1+k)\cdot R_p \cdot R_L}{R_L + (1+k)\cdot R_p} =$$

$$\frac{(1-k)\cdot R_p \cdot \left(R_L + (1+k)\cdot R_p\right) + (1+k)\cdot R_p \cdot R_L}{R_L + (1+k)\cdot R_p} \frac{(1-k)\cdot R_p \cdot R_L + \left(1-k^2\right)\cdot R_p^{\ 2} + (1+k)\cdot R_p \cdot R_L}{R_L + (1+k)\cdot R_p} =$$

$$\frac{2\cdot R_p \cdot R_L + \left(1-k^2\right)\cdot R_p^{\ 2}}{R_L + (1+k)\cdot R_p} = \frac{2\cdot R_p \cdot R_L \left(1 + \left(\frac{1-k^2}{2}\right)\cdot \frac{R_p}{R_L}\right)}{R_L + (1+k)\cdot R_p} = \frac{2\cdot R_p \cdot \left(1 + \left(\frac{1-k^2}{2}\right)\cdot \frac{R_p}{R_L}\right)}{1 + (1+k)\cdot \frac{R_p}{R_L}}$$

Circuit Analysis, Ohms Laws

4) … Solution

$$R_{eq} = \frac{2 \cdot R_p \cdot \left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right)}{1 + (1+k) \cdot \frac{R_p}{R_L}} \longrightarrow \boxed{I = \frac{e_i}{\mathrm{R}_{eq}}} \to\to e_i = e_0 + I \cdot (1-k) \cdot R_P =$$

$$e_0 + (1-k) \cdot R_P \cdot \frac{e_i}{\mathrm{R}_{eq}} \to\to e_0 = e_i \left(1 - \frac{(1-k) \cdot R_P}{\mathrm{R}_{eq}}\right)$$

Simplify …

$$e_0 = e_i \left(1 - \frac{(1-k) \cdot R_P}{\mathrm{R}_{eq}}\right) = e_i \left(1 - (1-k) \cdot R_P \frac{1 + (1+k) \cdot \frac{R_p}{R_L}}{2 \cdot R_p \cdot \left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right)}\right) =$$

$$e_i \left(1 - \frac{1}{2} \cdot \frac{1 - k + (1-k^2) \cdot \frac{R_p}{R_L}}{\left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right)}\right) = e_i \left(\frac{\left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right) - \frac{1-k}{2} + \frac{(1-k^2)}{2} \cdot \frac{R_p}{R_L}}{\left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right)}\right) = \frac{e_i}{2} \left(\frac{k+1}{\left(1 + \left(\frac{1-k^2}{2}\right) \cdot \frac{R_p}{R_L}\right)}\right)$$

Circuit Analysis, Ohms Laws

4) …

$$R_{L \Rightarrow \infty} \rightarrow e_0 = e_i \left(\frac{k+1}{2} \right)$$

$\rightarrow$ *Check….Voltage….Divider…*
for infinite impedance

$$\frac{e_o}{e_i} = \frac{R_p + k \cdot R_p}{(1-k) \cdot R_p + R_p + k \cdot R_p} = \frac{(1+k)}{2}$$

III. Error Analysis, Beckwith Chapter 3, Section 1.2 of Class Notes

~~1.4425~~ $k\Omega$

5) … Problem 3.3 in Beckwith

(a) A 68-kΩ resistor is paralleled with a 12-kΩ resistor. Each resistor has a ±10% tolerance. What will be the nominal resistance and the uncertainty of the combination?

(b) If the values remain the same except that the tolerance on the 68-kΩ resistor is dropped to ±5%, what will be the uncertainty of the combination?

Solution

a)

$R_1 = [68 \pm 6.8] \times 10^{-3}$

$R_2 = [12 \pm 1.2] \times 10^{-3}$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{12 \times 68}{12 + 68} = 10.2_{k\Omega}$$

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{12 \times 68}{68 + 12} \times 10^3$$

$= \underline{10.2 \, k\Omega}$

III. Error Analysis, Beckwith Chapter 3, Section 1.2 of Class Notes

5) … Problem 3.3 in Beckwith ***Solution***

$$U_{R_{eq}} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 {U_{R_1}}^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 {U_{R_2}}^2} \rightarrow$$

$$\frac{\partial R_{eq}}{\partial R_1} = \frac{R_2}{R_1+R_2} - \frac{R_1 \cdot R_2}{(R_1+R_2)^2} = \frac{R_2(R_1+R_2) - R_1 \cdot R_2}{(R_1+R_2)^2} = \left(\frac{R_2}{R_1+R_2}\right)^2$$

$$\frac{\partial R_{eq}}{\partial R_2} = \frac{R_1}{R_1+R_2} - \frac{R_1 \cdot R_2}{(R_1+R_2)^2} = \frac{R_1(R_1+R_2) - R_1 \cdot R_2}{(R_1+R_2)^2} = \left(\frac{R_1}{R_1+R_2}\right)^2$$

$$U_{R_{eq}} = \sqrt{\left(\frac{R_2}{R_1+R_2}\right)^4 {U_{R_1}}^2 + \left(\frac{R_1}{R_1+R_2}\right)^4 {U_{R_2}}^2} =$$

$$\left(\left(\frac{12}{12+68}\right)^4 6.8^2 + \left(\frac{68}{12+68}\right)^4 1.2^2\right)^{0.5} = \pm\, 0.880 k\Omega$$

III. Error Analysis, Beckwith Chapter 3, Section 1.2 of Class Notes

5) …

Problem 3.3 in Beckwith ***Solution***

$$U_{R_{eq}} = \sqrt{\left(\frac{R_2}{R_1+R_2}\right)^4 U_{R_1}{}^2 + \left(\frac{R_1}{R_1+R_2}\right)^4 U_{R_2}{}^2} =$$

$$\left(\left(\frac{12}{12+68}\right)^4 \left(\frac{6.8}{2}\right)^2 + \left(\frac{68}{12+68}\right)^4 (1.2)^2\right)^{0.5} \quad = \underline{+}\ 0.870k\Omega$$

III. Error Analysis

6) … Problem 3.6 in Beckwith

A 47-Ω resistor is connected in series with a parallel combination of a 100-Ω resistor and a 180-Ω resistor. What is the overall resistance of the array and what is the best estimate of its tolerance?

(a) For individual tolerances equal to 1%

(b) For tolerances of the 47-Ω resistors equal to 10% and for the 180-Ω resistors equal to 5%

Solution …

a)

$R_1 = 47\,\Omega \qquad u_{R_1} = .47\,\Omega$

$R_2 = 100\,\Omega \qquad u_{R_2} = 1\,\Omega$

$R_3 = 180\,\Omega \qquad u_{R_3} = 1.8\,\Omega$

$$R = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 111.286_{\Omega}$$

III. Error Analysis

6) … Problem 3.6 in Beckwith

(a) For individual tolerances equal to 1%

Solution … for part a)

$$\left[\begin{array}{c} \frac{\partial R}{\partial R_1} = 1 \\ \frac{\partial R}{\partial R_2} = \left(\frac{R_3}{R_2 + R_3}\right)^2 \\ \frac{\partial R}{\partial R_3} = \left(\frac{R_2}{R_2 + R_3}\right)^2 \end{array}\right] \rightarrow$$

$$U_R = \sqrt{\left(R_1 \cdot \frac{U_{R_1}}{R_1}\right)^2 + \left(\left(\frac{R_3}{R_2 + R_3}\right)^2 U_{R_2}\right)^2 + \left(\left(\frac{R_2}{R_2 + R_3}\right)^2 U_{R_3}\right)^2} =$$

$$\sqrt{\left(R_1 \cdot \frac{U_{R_1}}{R_1}\right)^2 + \left(\left(\frac{R_2 \cdot R_3}{R_2 + R_3}\right)^2 \cdot \frac{1}{R_2} \cdot \frac{U_{R_2}}{R_2}\right)^2 + \left(\left(\frac{R_2 \cdot R_3}{R_2 + R_3}\right)^2 \cdot \frac{1}{R_3} \cdot \frac{U_{R_3}}{R_3}\right)^2} =$$

$$\sqrt{(47_\Omega \cdot 0.01)^2 + \left((64.2857_\Omega)^2 \cdot \frac{1}{100_\Omega} \cdot 0.01\right)^2 + \left((64.2857_\Omega)^2 \cdot \frac{1}{180_\Omega} 0.01\right)^2} =$$

$$\pm 0.667_\Omega$$

III. Error Analysis

6) … Problem 3.6 in Beckwith

Solution … for part b)

(b) For tolerances of the 47-Ω resistors equal to 10% and for the 180-Ω resistors equal to 5%

$$\left[\begin{array}{c} \frac{\partial R}{\partial R_1}=1 \\ \frac{\partial R}{\partial R_2}=\left(\frac{R_3}{R_2+R_3}\right)^2 \\ \frac{\partial R}{\partial R_3}=\left(\frac{R_2}{R_2+R_3}\right)^2 \end{array}\right] \rightarrow$$

$$U_R=\sqrt{\left(R_1\cdot\frac{U_{R_1}}{R_1}\right)^2+\left(\left(\frac{R_3}{R_2+R_3}\right)^2 U_{R_2}\right)^2+\left(\left(\frac{R_2}{R_2+R_3}\right)^2 U_{R_3}\right)^2}=$$

$$\sqrt{\left(R_1\cdot\frac{U_{R_1}}{R_1}\right)^2+\left(\left(\frac{R_2\cdot R_3}{R_2+R_3}\right)^2\cdot\frac{1}{R_2}\cdot\frac{U_{R_2}}{R_2}\right)^2+\left(\left(\frac{R_2\cdot R_3}{R_2+R_3}\right)^2\cdot\frac{1}{R_3}\cdot\frac{U_{R_3}}{R_3}\right)^2}=$$

$$\sqrt{(47_\Omega\cdot 0.1)^2+\left((64.2857_\Omega)^2\cdot\frac{1}{100_\Omega}\cdot 0.01\right)^2+\left((64.2857_\Omega)^2\cdot\frac{1}{180_\Omega}0.05\right)^2}=$$

$$\pm 4.856_\Omega$$

IV. Signal and Fourier Series, Beckwith Chapter 4, Sections 2.1,2.2 of Class Notes

7) …Problem 4.1 in Beckwith

The following expression represents the displacement of a point as a function of time:

$$y(t) = 100 + 95 \sin 15t + 55 \cos 15t$$

(a) What is the fundamental frequency in hertz?

(b) Rewrite the equation in terms of cosines only.

$y(t) = 100 + 95 \sin 15t + 55 \cos 15t$

a) $\omega = 2\pi f = 15$ rad/s

$f = \frac{15}{2\pi} = 2.39$ Hz

b) $C = \sqrt{A^2 + B^2} = \sqrt{55^2 + 95^2} = 109.77$

$\phi = \tan^{-1} \frac{B}{A} = 1.046$ rad

$y(t) = 100 + 109.77 \cos(15t - 1.046)$

Signal and Fourier Series

8) …Problem 4.9 in Beckwith

a) What is the fundamental frequency of the waveform below?

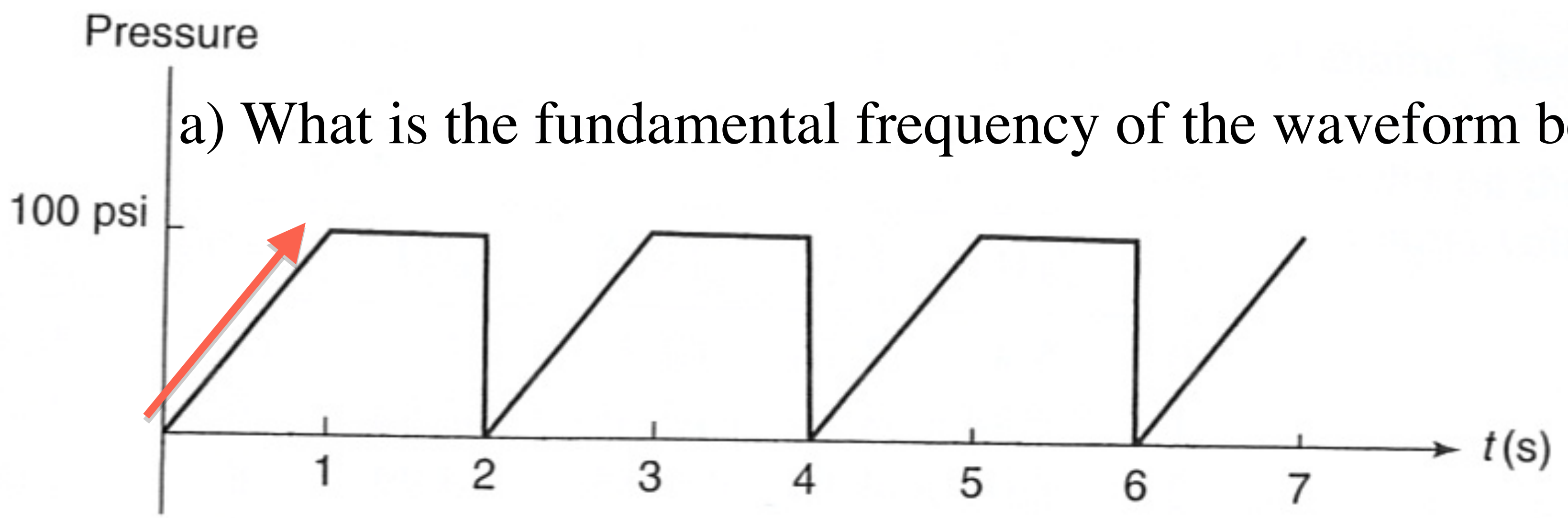


FIGURE 4.23: Pressure–time record for Problem 4.9.

b) Calculate the mean value and Fourier Series Expansion of the signal?

Signal and Fourier Series

8) …Problem 4.9 in Beckwith (Solution)

$$T = 2.0\ s \qquad 0 \qquad f = \frac{1}{T} = .5\ Hz$$

$$0\ \omega = \pi \text{ rad/s}$$

$$A_0 = \frac{2}{T}\int_0^1 100t\,dt + \frac{2}{T}\int_1^2 100\,dt$$

Fundamental Frequency

$$A_0 = 150\text{ psi} \qquad \frac{A_0}{2} = 75\text{ psi}$$

Mean Value

$$A_n = \frac{2}{T}\int_0^1 100t\cos n\pi t\,dt + \frac{2}{T}\int_1^2 100\cos n\pi t\,dt$$

Signal and Fourier Series

8) …Problem 4.9 in Beckwith (Solution)

$$A_n = \frac{2}{T}\left[\int_0^1 100\cdot t\cdot\cos n\cdot\pi\cdot t\cdot dt + \int_1^2 100\cdot\cos n\cdot\pi\cdot t\cdot dt\right]$$

$$\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int_0^1 t\cdot\cos n\cdot\pi\cdot t\cdot dt = \frac{1}{(n\cdot\pi)^2}\cdot(\cos n\cdot\pi\cdot 1 - \cos n\cdot\pi\cdot 0) + \frac{\sin n\cdot\pi}{(n\cdot\pi)} - (0) = \frac{(-1)^n - 1}{(n\cdot\pi)^2}$$

$$\int_1^2 \cos n\cdot\pi\cdot t\cdot dt = \frac{1}{(n\cdot\pi)}(\sin n\cdot\pi\cdot 2 - \sin n\cdot\pi\cdot 1) = 0$$

$$A_n = \frac{200}{T}\cdot\frac{(-1)^n - 1}{(n\cdot\pi)^2} = 100\cdot\frac{(-1)^n - 1}{(n\cdot\pi)^2} \Longrightarrow$$

$$n = \{1,\quad 2,\quad 3,\quad 4,\quad 5,\quad 6\ldots\}$$

$$A_n = \left\{-\frac{200}{\pi^2}, 0, -\frac{200}{(3\cdot\pi)^2}, 0, -\frac{2000}{(5\cdot\pi)^2}, 0\ldots\right\}$$

Signal and Fourier Series

8) …Problem 4.9 in Beckwith (Solution)

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax$$

$$\int x \sin ax\,dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2}$$

$$B_n = \frac{2}{T}\left[\int_0^1 100\cdot t\cdot \sin n\cdot\pi\cdot t\cdot dt + \int_1^2 100\cdot \sin n\cdot\pi\cdot t\cdot dt\right]$$

$$\int_0^1 t\cdot\sin n\cdot\pi\cdot t\cdot dt = \frac{-1}{(n\cdot\pi)}\cdot(1\cdot\cos n\cdot\pi\cdot 1 - 0\cdot\cos n\cdot\pi\cdot 0) + \frac{\sin n\cdot\pi\cdot 1}{(n\cdot\pi)^2} - \frac{\sin n\cdot\pi\cdot 0}{(n\cdot\pi)^2} = -\frac{(-1)^n}{(n\cdot\pi)}$$

$$\int_1^2 \sin n\cdot\pi\cdot t\cdot dt = \frac{-1}{(n\cdot\pi)}(\cos n\cdot\pi\cdot 2 - \cos n\cdot\pi\cdot 1) = \frac{-1}{(n\cdot\pi)}\left(1-(-1)^n\right) = \frac{(-1)^n - 1}{(n\cdot\pi)}$$

$$B_n = \frac{200}{T}\cdot\left[-\frac{(-1)^n}{(n\cdot\pi)} + \frac{(-1)^n - 1}{(n\cdot\pi)}\right] = \frac{-100}{(n\cdot\pi)}$$

$$n = \{1,\quad 2,\quad 3,\quad 4,\quad 5,\quad 6\}\ldots$$

$$\rightarrow \quad B_n = \left\{\frac{-100}{\pi}, \frac{-100}{2\pi}, \frac{-100}{3\pi}, \frac{-100}{4\pi}, \frac{-100}{5\pi}, \frac{-100}{6\pi}\ldots\right\}$$

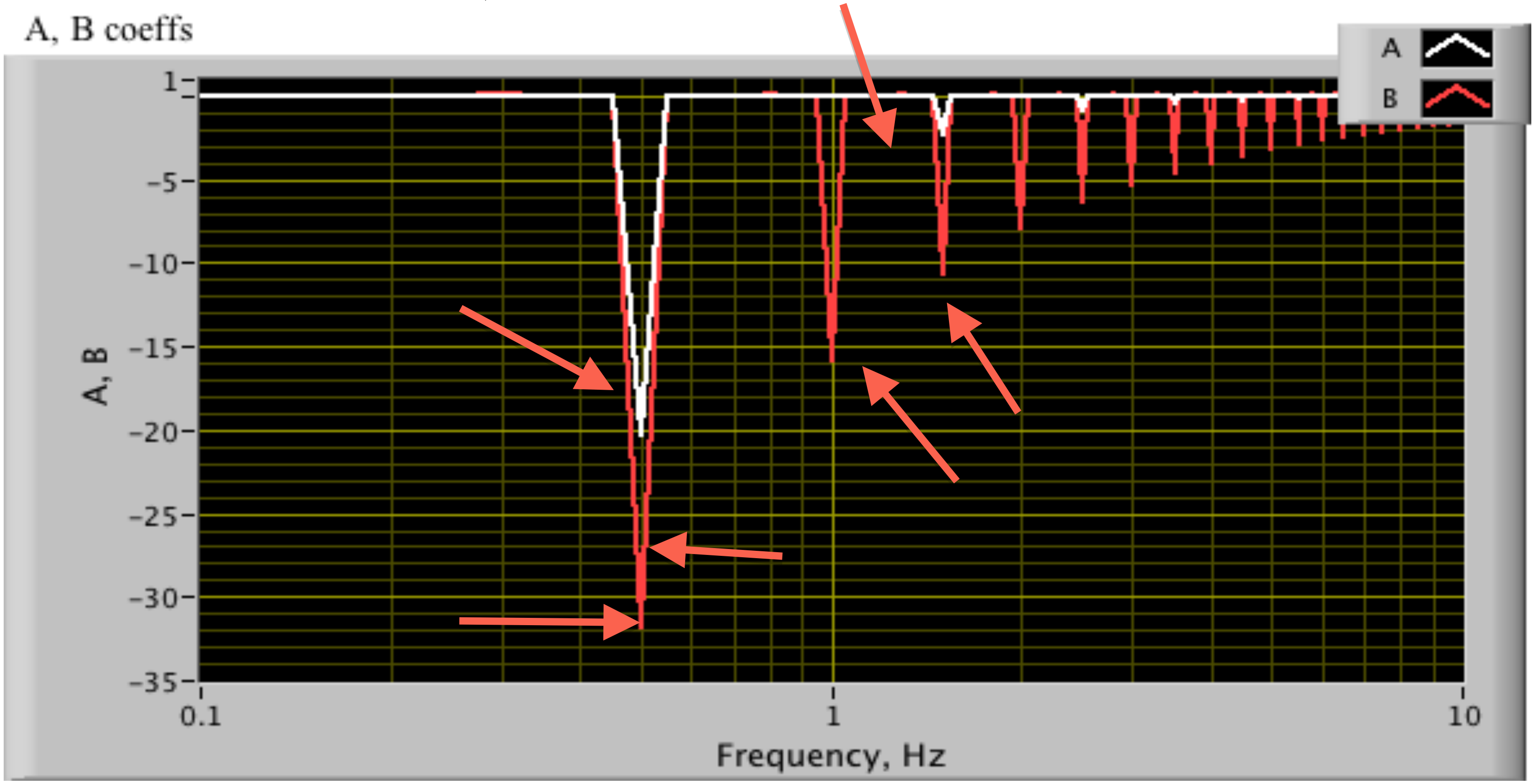
$$\cos(n\cdot 2\cdot\pi) = 1 \ for\ n = \{1,2,\ldots,\infty\}$$

$$n = 1 \rightarrow \cos(1\cdot\pi) = -1$$
$$n = 2 \rightarrow \cos(2\cdot\pi) = +1$$
$$n = 3 \rightarrow \cos(3\cdot\pi) = -1$$
$$n = 4 \rightarrow \cos(4\cdot\pi) = +1$$
$$\ldots$$
$$\rightarrow \cos(n\cdot\pi) = (-1)^n$$

Signal and Fourier Series

8) …Problem 4.9 in Beckwith (Solution)

$$y(t) = 75_{psi} + \sum_{n=1}^{\infty}\left(100\cdot\frac{(-1)^n - 1}{(n\cdot\pi)^2}\cdot\cos n\cdot\pi\cdot t + \frac{-100}{(n\cdot\pi)}\cdot\sin n\cdot\pi\cdot t\right)_{psi}$$



Signal and Fourier Series

9) …Problem 4.13 in Beckwith

A 500-Hz sine wave is sampled at a frequency of 4096 Hz. A total of 2048 points are taken.

(a) What is the Nyquist frequency?

(b) What is the frequency resolution?

$$f_{Nyq} = \frac{1}{2 \cdot \Delta t} = \frac{fs}{2} = \frac{4096\,Hz}{2} = 2048\,Hz$$

$$T_{record} = N \cdot \Delta t = \frac{N}{f_s} = \frac{2048}{4096} = \frac{1}{2}\ seconds$$

$$\rightarrow \Delta f_{\min} = \frac{1}{T_{record}} = 2\ Hz \ldots \text{"}Bandwidth\ resolution\text{"}$$

Signal and Fourier Series

9) …Problem 4.13 in Beckwith

(c) The student making the measurement suspects that the sampled waveform contains several harmonics of 500 Hz. Which of these can be accurately measured? What happens to the others?

Harmonics … *n=1 (500 Hz), n=2 (1000 Hz), n=3 (1500 Hz)*

n=4 (2000 Hz) can be captured … higher harmonics get aliased

n=5 (2500 Hz) … aliased to *2048 Hz – (2500-2048) = 1596 Hz*

n=6 (3000 Hz) … aliased to *2048 Hz – (3000-2048) = 1096 Hz*

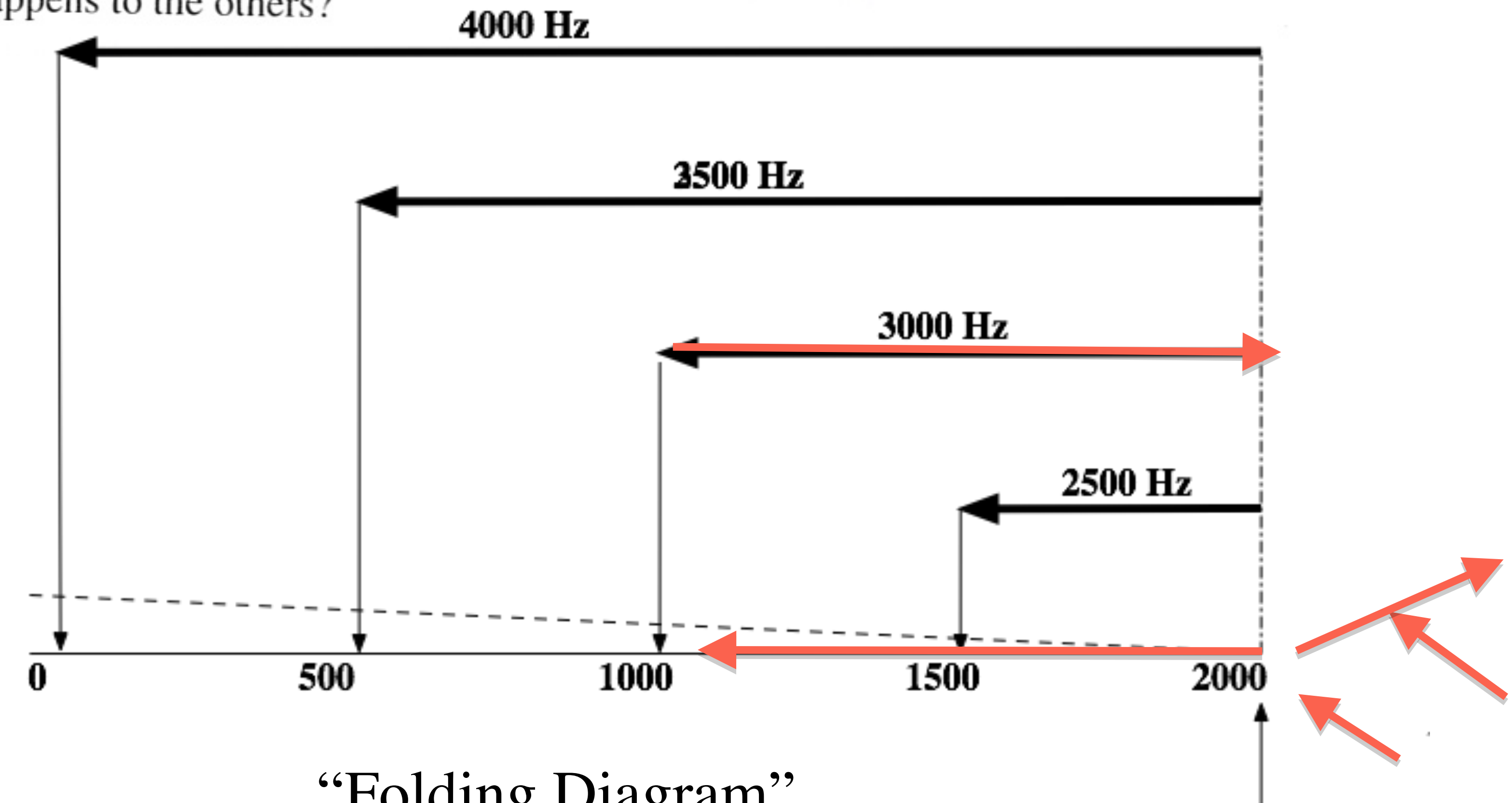
n=7 (3500 Hz) … aliased to *2048 Hz – (3500-2048) = 596 Hz*

n=8 (4000 Hz) … aliased to *2048 Hz – (4000-2048) = 96 Hz*

Signal and Fourier Series

9) …Problem 4.13 in Beckwith

(c) The student making the measurement suspects that the sampled waveform contains several harmonics of 500 Hz. Which of these can be accurately measured? What happens to the others?



"Folding Diagram"

$f_{Nyq} = 2048\,Hz$

Signal and Fourier Series

10) …Example 4. 3 in Beckwith

Helium-neon laser light has a frequency of 473.8 THz (473.8×10^{12} Hz). A helium-neon laser beam is reflected from a moving target. This creates a doppler shift in the beam, which increases its frequency by 3 MHz (3×10^6 Hz).[1] The reflected beam is "added to" an unshifted beam of equal intensity, by using mirrors to bring the beams together. What is the resulting signal?

Solution From Eq. (4.9), with $f_0 = 473.8$ THz and $f_1 = f_0 + \Delta f = 473.8$ THz$+3$ MHz,

$$y = 2A\cos\left(2\pi\frac{\Delta f}{2}t\right)\sin\left(2\pi\frac{f_0+f_1}{2}t\right)$$

$$= 2A\cos\left[2\pi(1.5\times10^6)t\right]\sin\left[2\pi(473.8\times10^{12})t\right]$$

The amplitude has a cyclic frequency of 1.5×10^6 Hz $=$ 1.5 MHz, causing zeros in amplitude twice per cycle for a beat frequency of 3-MHz. This would be manifested

V. Instrumentation, Calibration, and Resolution, , Beckwith Chapter 1, Sections 1.1, 1.2 of Class Notes

PXCPC-150GV In Stock $35.00 150 psig miniature pressure sensor

Specifications
@ 12 Vdc, 25°C (77°F)
Excitation: 12 Vdc typical (3 Vdc minimum to 16 Vdc maximum)
Output mV (@ 12 Vdc) Range:
- **4 inH2O:** 25 ±2 mV
- **10 inH2O:** 20 ±1 mV
- **1 psi:** 18 ±1 mV
- **5 psi:** 60 ±1 mV
- **15 to 60 psi:** 90 ±5 mV
- **100 psi:** 100 ±5 mV
- **150 psi:** 90 ±5 mV

Linearity, Hysteresis Error: 0.25% typical 1% maximum
Operating Temperature: -25 to 85°C (-13 to 185°F)

Miniature Silicon Pressure Sensors, Temperature compenstate

- Consider the Pressure Transducer PXCPC-150GV
- 0-150 psig full scale (above ambient) gives *0-90 mV* output
- ±1% Linearity, Hysteresis Error (maximum)
- ***A gauge pressure transducer measured the deviation from the local ambient pressure***

11) Instrumentation, Calibration and Resolution

- Consider the Pressure Transducer PXCPC-150GV
- 0-150 psig full scale (above ambient) gives *0-90 mV* output
- +1% Linearity, Hysteresis Error (maximum) ***Volt/psig (output/input)?***

a) … what is the nominal sensitivity of the transducer in units of Volts/psig (output/input)

$$Sensititivy = \frac{90_{mV}}{150_{psig}} \times \frac{1_V}{1000_{mV}} = 0.006\ \frac{V}{psig}$$

b) … If the anticipated nominal pressure level for your experiment is 1000 kPa gauge, what output voltage would you expect from your transducer?

$$Output = 0.006\ \frac{V}{psig} \times 1000_{kPa} \times \frac{14.696\ \frac{psig}{kPa-g}}{101.325} = 0.87_{Volts}$$

c) Based on the spec … 150 psig full scale = *90 mV* $\pm$ *5 mV* output … What is the expected accuracy of your (1000 kPa) measurement in units of kPa? …. percentage of reading?

11) Instrumentation, Calibration and Resolution

c) Based on the spec … 150 psig full scale = *90 mV* ± *5 mV* output … ***What is the expected precision uncertainty of your measurement in terms of the percentage of full scale reading?***

$$\frac{U_{precision}}{P_{full/scale}} = \pm\frac{5_{mV}}{90_{mV}} \times 100 = 5.56\%_{full/scale}$$

In units of kPa? … assuming a nominal pressure input of 1000 kPa? psig?

$$Full/scale_{kPa} = 150_{psi}g \times \frac{101.325_{\frac{kPa-g}{psig}}}{14.696} = 1034.21_{kPa}$$

$$U_{precision\,kPa} = \pm\frac{5.56\%}{100} \times 1000_{kPa} = \pm 55.6 kPa$$

$$U_{precision\,kPa} = \pm 55.6 kPa \times \frac{14.696_{\frac{psig}{kPa-g}}}{101.325} = \pm 8.067_{psig}$$

11) Instrumentation, Calibration and Resolution

d) You are going to sample the transducer output using one of two available USB-Based DAQ devices

NI-6009 USB-Based DAQ,
8 analog inputs (14-bit resolution)

- **Which Unit Should you Chose? To give the best Resolution?**

NI USB-6002 ***Low-Cost Multifunction DAQ for Basic, Quality Measurements,***
8 analog inputs (16-bit resolution)

Input range..±10 V

Input range

Differential...±20 V[2], ±10 V, ±5 V, ±4 V, ±2.5 V, ±2 V, ±1.25 V, ±1 V

11) Instrumentation, Calibration and Resolution

- ***Calculate the available least-significant bit (lsb)for NI-6009 and USB-6002 and compare***

- ***LSB value in Volts/count, psig/count, kPa/count***

$$NI-6009 \rightarrow select \pm 1V \;\; minimum\ range$$

$$lsb \rightarrow \frac{2V}{2^{14}} = 0.12\times10^{-3}\ {}_{V/bit}$$

14-bit

$$U_{psig} \rightarrow \frac{0.12\times10^{-3}\ {}_{V/bit}}{0.006\ \frac{V}{psig}} = 0.0204\ \frac{psig}{count}$$

$$U_{kPa} \rightarrow 0.0204\ \frac{psig}{count} \times \frac{101.325\ \frac{kPa-g}{psig}}{14.696} = 0.140\ \frac{kPa}{count}$$

$$USB-6002 \rightarrow \pm 10V \;\; fixed\ range$$

$$lsb \rightarrow \frac{20V}{2^{16}} = 0.305\times10^{-3}\ {}_{V/bit}$$

16-bit

$$U_{psig} \rightarrow \frac{0.12\times10^{-3}\ {}_{V/bit}}{0.006\ \frac{V}{psig}} = 0.051\ \frac{psig}{count}$$

$$U_{kPa} \rightarrow 0.051\ \frac{psig}{count} \times \frac{101.325\ \frac{kPa-g}{psig}}{14.696} = 0.351\ \frac{kPa}{count}$$

14-Bit DAQ with $\underline{+}$1V range has better selection for this application

11) Instrumentation, Calibration and Resolution

- ***How many "counts" will your DAQ display for the nominal 1000 kPa reading?***

$$\rightarrow \frac{1000_{kPa}}{0.14\ \frac{kPa}{count}} = 7142\ counts$$

- ***What is your estimated end-to-end measurement accuracy Include precision, hysteresis, resolution of DAQ?***

$$U_{precision\,kPa} = \pm 55.6_{kPa}$$

$$U_{resolution\ kPa} = \pm 0.140\ \frac{kPa}{count}$$

$$U_{hysteresis} = \pm 1\%_{fullscale} = \pm 0.01 \times 150_{psig} \times \frac{101.325\ \frac{kPa}{psig}}{14.696} = 10.342_{kPa}$$

$$U_{total} = \sqrt{U^2_{precision\,kPa} + U^2_{resolution\ kPa} + U^2_{hysteresis}} = \sqrt{55.6^2 + 0.14^2 + 10.342^2}_{kPa} = \pm 56.55_{kPa}$$

... or about 57% of expected reading

VI. Probability and Statistical Analysis

12) Problem 3.20 Beckwith

Results from a chemical analysis for the carbon content of two materials are as follows:

Carbon Content, %

Material A	93.52	92.81	94.32	93.77	93.57	93.12
Material B	92.38	93.21	92.55	92.05	92.54	

Determine if there is a significant difference in carbon content at the 99% confidence level.

VI. Probability and Statistical Analysis

12) Problem 3.20 Beckwith

prob 3.20	Data Set 1	Data Set 2
# Samples	6	5
Raw Data		
1	93.52	92.38
2	92.81	93.21
3	94.32	92.55
4	93.77	92.05
5	93.57	92.54
6	93.12	
7		
8		
9		
10		
11		
12		
13		
14		
15		
Average	93.5183333	92.546
Standard Dev	0.52327494	0.42264642
Eq. 3.25a Num	0.00661979	
3.25a Denom	0.00073562	
Nu	8	
t	3.40882001	

	A	B
n	6	5
$\bar{x}$	93.5	92.5
S_x	0.52	0.42

$\alpha = 1 - .99 = .01$

Eqn 3.25a $\nu = 8$

Eqn 3.25 $t = 3.409$

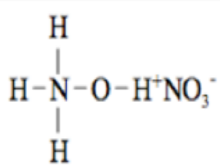
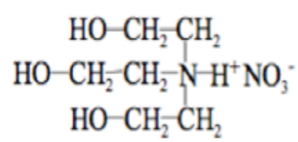
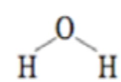
$t_{\alpha/2,\nu} = t_{.005,8} = 3.355$ (Table 3.6)

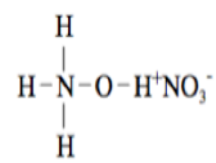
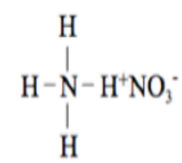
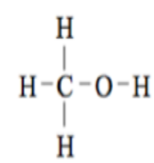
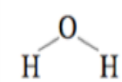
Since $t > 3.355$ → Significant Difference.

13) Propellant Analysis

• We've Been asked to compare the performance of two HAN-based propellant formulations

1. ... The well known Army-Developed HAN *LP 1846 HAN-Based* Monopropellant for Artillery and Ordnance Applications that consists of the mass proportions shown by the table below

LP 1846			
	HAN	TEAN	Water
Molecular Structure			
Wt. %	60.8	19.2	20.0

HAN269MEO15				
	HAN	AN (Ammonium Nitrate)	Methanol	Water
Molecular Structure				
Wt. %	69.7	0.6	14.79	14.91

2. .. The lesser-known known Aerojet Formulation HAN269MEO15 which consists of the mass proportions shown by the table above

Propellant Tests

The following results were collected for 8 HAN LP 1846 thruster tests

Test Results, HAN LP 1846

Test #	P0, kPa	F, N	Mdot , g/s	Isp, s
1	3085.72	37.4	29.61	151.54
2	2994.65	36.12	28.64	151.77
3	3150.54	38.31	29.94	153.45
4	3021.96	36.51	29.01	151.54
5	2962.97	35.69	28.71	149.83
6	2946.36	35.47	27.8	154.26
7	2947.05	35.49	28.1	153.12
8	2857.94	34.12	29.19	139.75
Mean	2995.9	36.1396	28.8768	150.659
Std Dev	90.7273	1.28611	0.718911	4.618
Conf. Interval	75.838	1.07504	0.60093	3.86013

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Estimate of the mean

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}[x_i - \bar{x}]^2}$$

Estimate of the Standard Deviation

$$\bar{x} - t_{c/2,\nu}\cdot\frac{S_x}{\sqrt{n}} \le \mu_x \le \bar{x} + t_{c/2,\nu}\cdot\frac{S_x}{\sqrt{n}}$$

$$t_{c/2=0.475,\nu=7} = 2.363$$

D.O.F =7

confidence interval $\rightarrow t_{c/2=0.475,\nu=7}\cdot\frac{S_x}{\sqrt{n}} = 2.363\cdot\frac{4.618}{\sqrt{8}} = 3.86$

$$\mu_{Isp} = 150.659 \pm 3.86\,\text{sec}$$

Results Analysis (cont'd)

The following test results were collected for HAN 269MEO15:

Test Results, HAN 269MEO15

Test #	P0, kPa	F, N	Mdot , g/s	Isp, s
1	3131.72	38.08	29.61	154.72
2	3066.18	37.19	28.63	156.67
3	3204.74	39.13	29.97	157.16
4	3065.72	37.16	29.01	154.55

Mean Values

3117.09	37.8904	29.303	155.774

Std Dev. Values

66.1503	0.92865	0.599395	1.33276

Conf. Interval

105.188	1.47668	0.953119	2.11927

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Estimate of the mean

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}[x_i - \bar{x}]^2}$$

Estimate of the Standard Deviation

$$\bar{x} - t_{c/2,\nu}\cdot\frac{S_x}{\sqrt{n}} \le \mu_x \le \bar{x} + t_{c/2,\nu}\cdot\frac{S_x}{\sqrt{n}}$$

$$t_{c/2=0.475,\nu=3} = 3.182$$

D.O.F =3

confidence interval $\rightarrow t_{c/2=0.475,\nu=3}\cdot\frac{S_x}{\sqrt{n}} = 3.182\cdot 1.332 = 2.119$

- ***Only 4 HAN 269MEO tests were performed due to limited propellant availability***

$$\mu_{Isp} = 154.55 \pm 2.119\,\text{sec}$$

Results Analysis (cont'd)

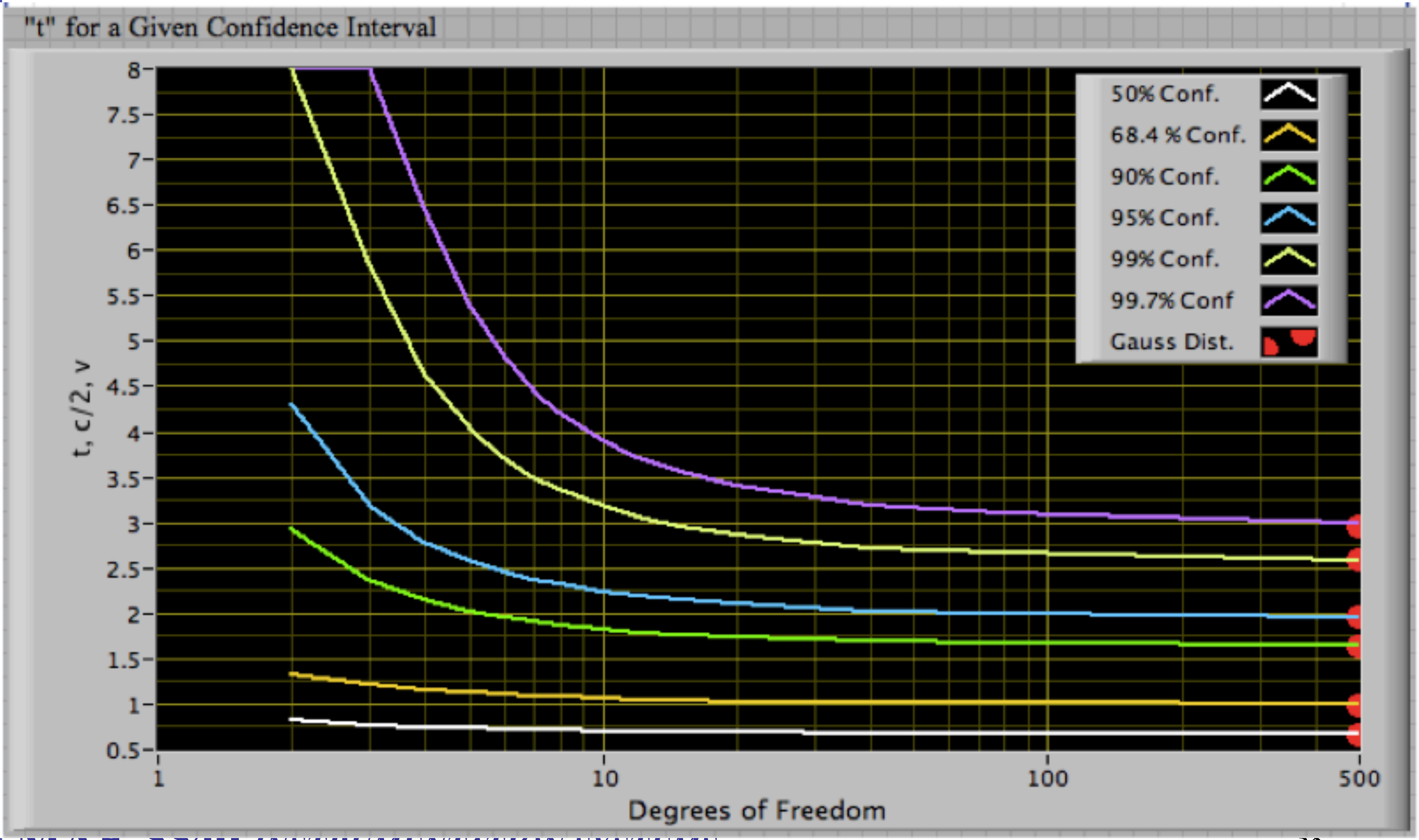
• Based on these test results, **Aerojet claims HAN269MEO15 generates superior performance when compared to LP 1846 … let's evaluate that claim!**

LP 1846	**HAN269MEO15**
$\mu_{Isp} = 150.659 \pm 3.86\,\mathrm{sec}$	$\mu_{Isp} = 154.55 \pm 2.119\,\mathrm{sec}$

• Use the *"t-test"* and the appropriate degrees of freedom to assess whether the observed differences in Specific Impulse are statistically significant.

• Use a 95% confidence interval for this assessment.

Results Analysis (cont'd)



Results Analysis (solution)

- *Calculate t-variable*

$$t = \frac{\bar{x}_{269\,MEO} - \bar{x}_{LP1846}}{\sqrt{\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}}} = \frac{155.774 - 150.659}{\left(\left(\frac{1.33276^2}{4}\right) + \left(\frac{4.618^2}{8}\right)\right)^{0.5}} = 2.90054$$

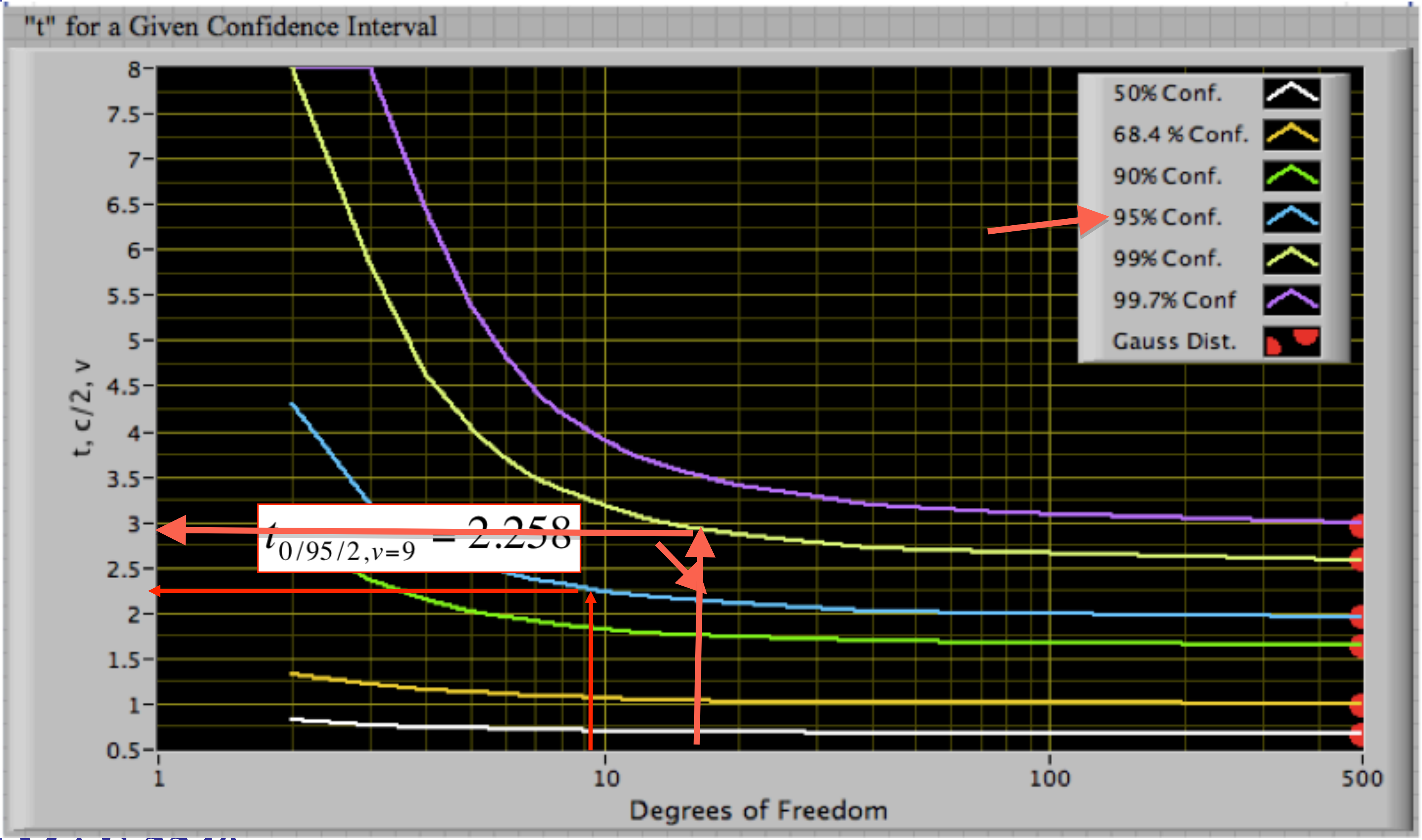
sec

- *Calculate effective DOF*

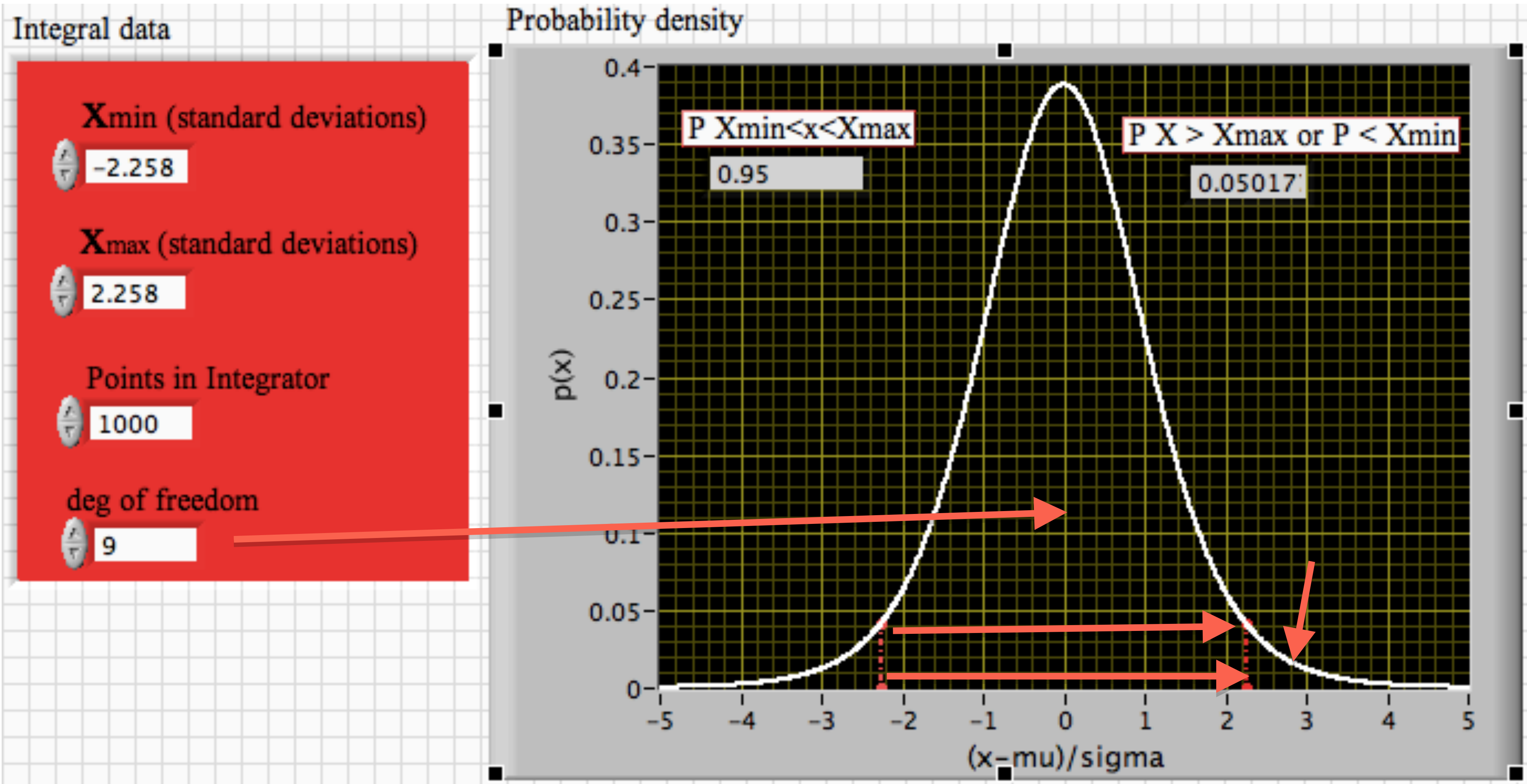
$$v_{avg} = \frac{\left[\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}\right]^2}{\left[\frac{\left(\frac{S^2_{269\,MEO}}{n_{269\,MEO}}\right)^2}{n_{269\,MEO} - 1} + \frac{\left(\frac{S^2_{LP1846}}{n_{LP1846}}\right)^2}{n_{LP1846} - 1}\right]^2} = \frac{\left(\left(\frac{1.33276^2}{4}\right) + \left(\frac{4.618^2}{8}\right)\right)^2}{\frac{\left(\frac{1.33276^2}{4}\right)^2}{4-1} + \frac{\left(\frac{4.618^2}{8}\right)^2}{8-1}} = 8.94707$$

Round to 9 DOF

Results Analysis (solution)



Results Analysis (solution)



Results Analysis (solution)

$$t = \frac{\bar{x}_{269\,MEO} - \bar{x}_{LP1846}}{\sqrt{\frac{S^2_{269\,MEO}}{n_{269\,MEO}} + \frac{S^2_{LP1846}}{n_{LP1846}}}} = 2.90054 > t_{c/2,\,v} = 2.258$$

…. Performance of HAN 269MEO15 is statistically better than HAN LP1846 at a 95% confidence level

…. In fact the statistical difference is valid up to a 98.2% confidence level!

14) Problem 3.24 Beckwith

Viscosity (Dimensionless)

Sample No.	Using Apparatus A	Using Apparatus B
1	72	73
2	43	45
3	54	56
4	75	75
5	50	53
6	48	50
7	73	72
8	55	54
9	48	48
10	50	52

	A	B
n	10	10
$\bar{x}$	56.8	57.8
S_x	11.9	11.17

$\nu = 17$

$t = -0.194$

$\alpha = 1 - .99 = .01$

$t_{\alpha/2,\nu} = t_{.005,17} = 2.898$ Table 3.6

Determine whether there is a significant difference in the two systems at the 99% confidence level.

Since $|t| < 2.898$ → No Significant Difference

14) Problem 3.24 Beckwith

prob 3.24	Data Set 1	Data Set 2
# Samples	10	10
Raw Data		
1	72	73
2	43	45
3	54	56
4	75	75
5	50	53
6	48	50
7	73	72
8	55	54
9	48	48
10	50	52
11		
12		
13		
14		
15		
Average	56.8	57.8
Standard Dev	11.8958443	11.1733811
Eq. 3.25a Num	709.45282	
3.25a Denom	39.5683665	
Nu	17	
t	-0.1937622	

	A	B
n	10	10
$\bar{x}$	56.8	57.8
S_x	11.9	11.17

$\nu = 17$

$t = -0.194$

$\alpha = 1 - .99 = .01$

$t_{\alpha/2,\nu} = t_{.005,17} = 2.898$ Table 3.6

Since $|t| < 2.898$ → No Significant Difference